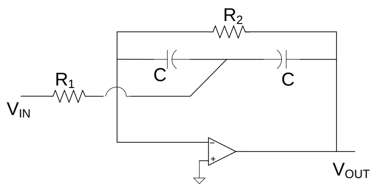
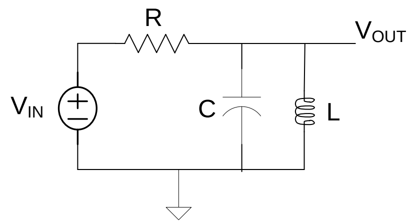
EE 508
Lecture 18

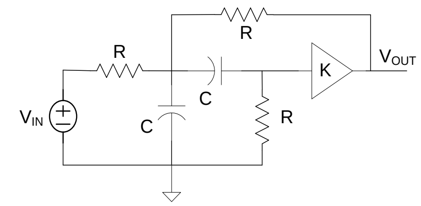
Sensitivity Functions

- Comparison of Filter Structures
- Performance Prediction
- Design Characterization

How does the performance of these bandpass filters compare?

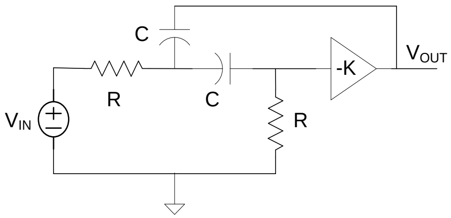


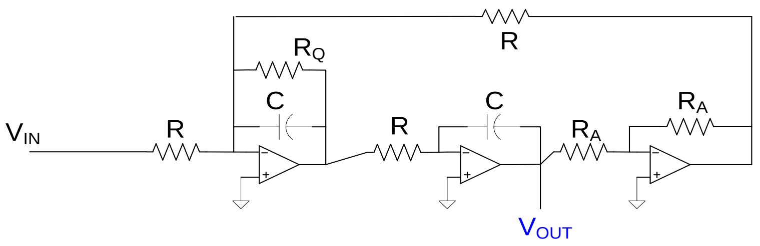




Review from last time

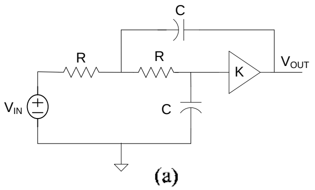
- Component Spread
- Number of Op Amps
- Is the performance strongly dependent upon how DOF are used?
- Ease of tunability/calibration (but practical structures often are not calibrated)
- Total capacitance or total resistance
- Power Dissipation
- Sensitivity
- Effects of Op Amps

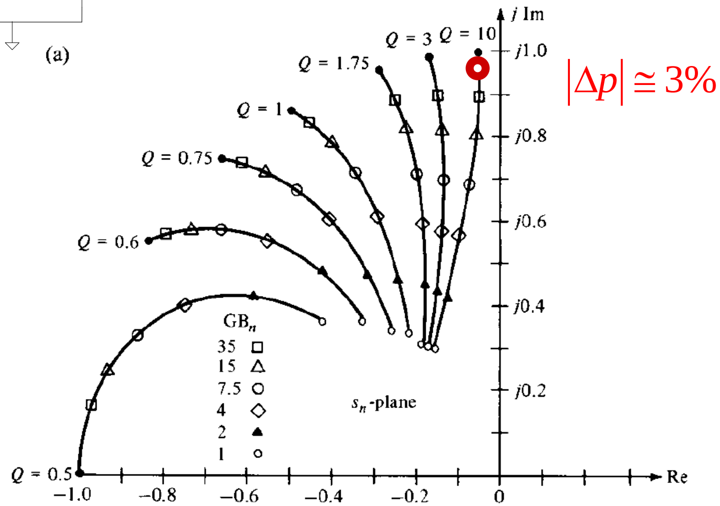


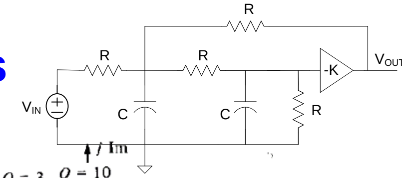


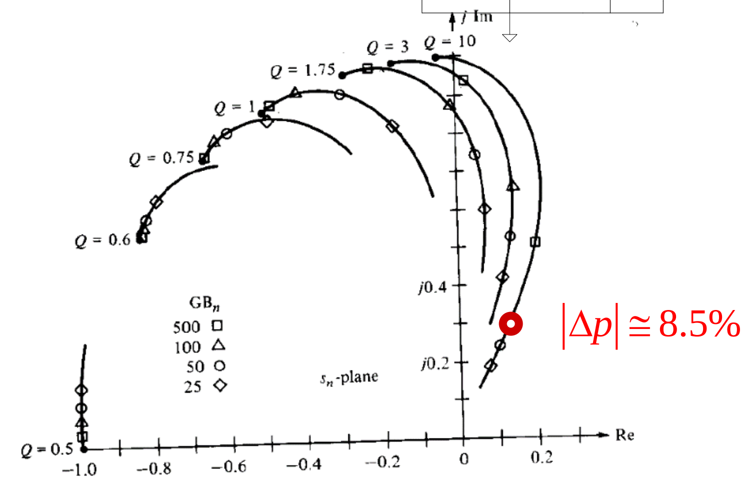
Comparison of 4 second-order LP filters

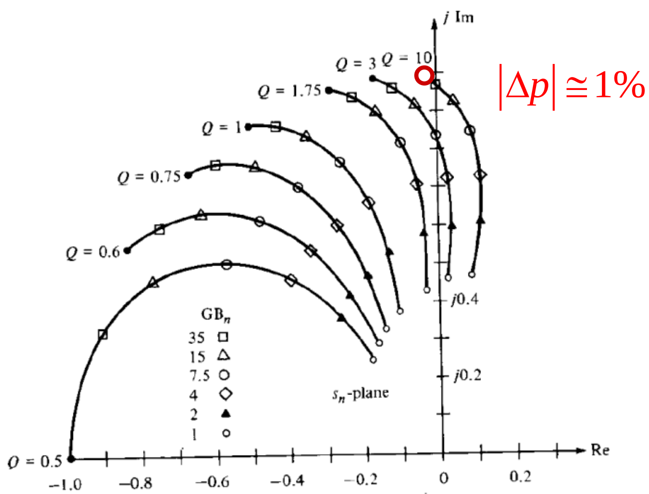
Review from last time

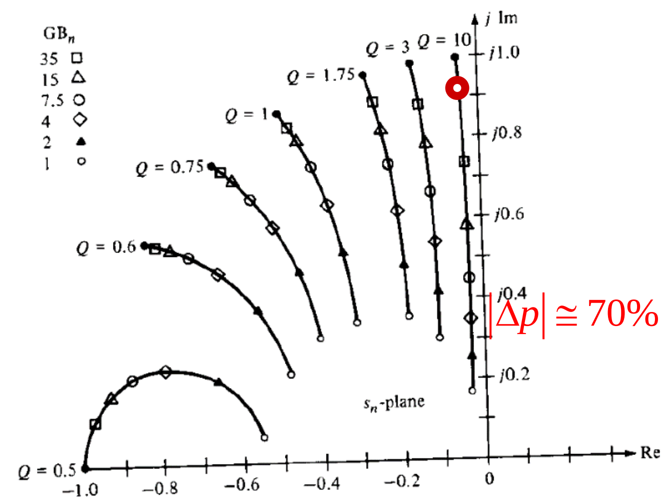


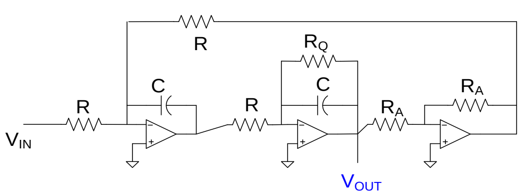




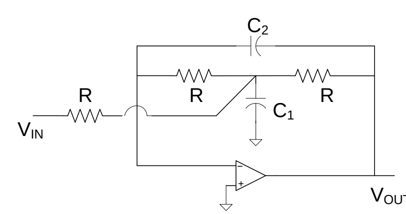








consider ⊙ ⟷ $GB_n = \frac{GB}{\omega_0} = 100$



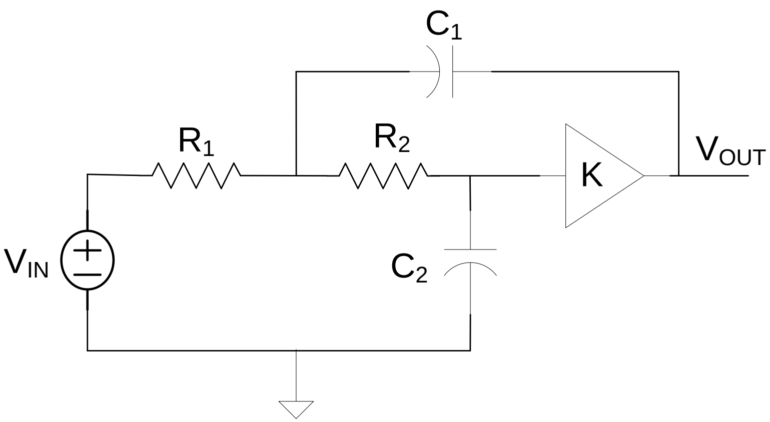
Some Observations

Review from last time

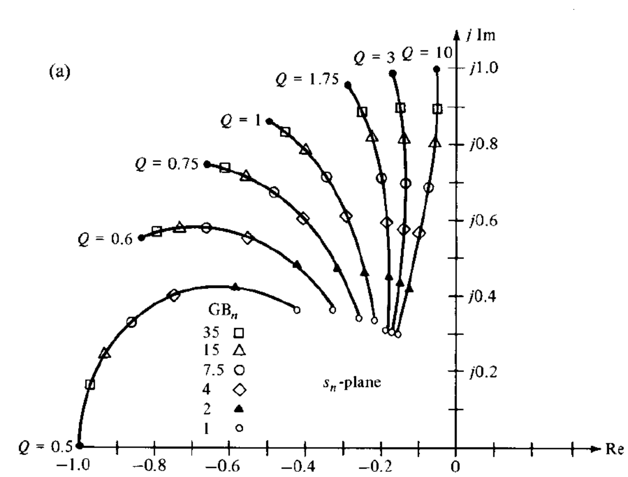
- Seemingly similar structures have dramatically different sensitivity to frequency response of the Op Amp
- Critical to have enough GB if filter is to perform as desired
- Performance strongly affected by both magnitude and direction of pole movement
- Some structures appear to be totally impractical – at least for larger Q
- Different use of the Degrees of Freedom produces significantly different results

Sensitivity analysis is useful for analytical characterization of the performance of a filter

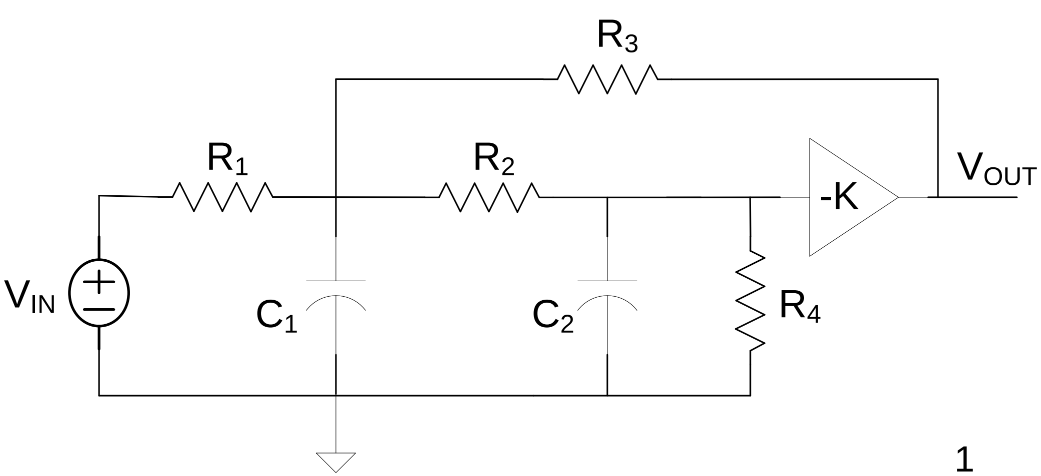
What causes the dramatic differences in performance between these two structures?
How can the performance of different structures be compared in general?

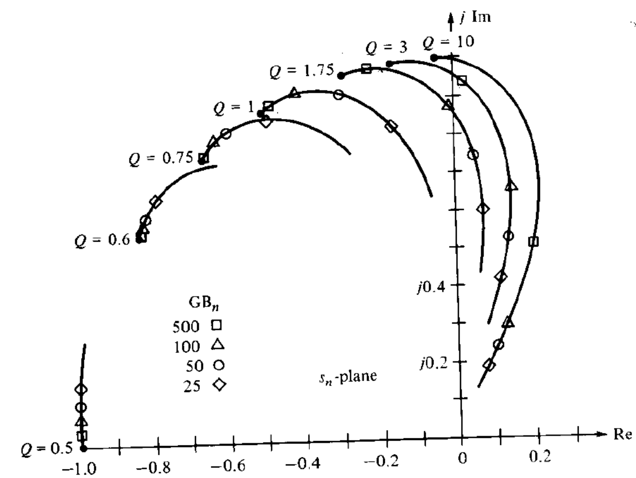


$$T(s)=K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$



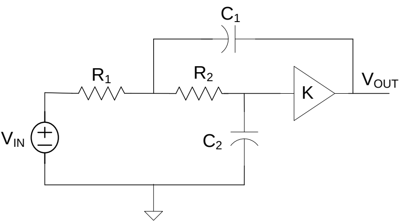
Equal R, Equal C, Q=10 Pole Locus vs GB_N

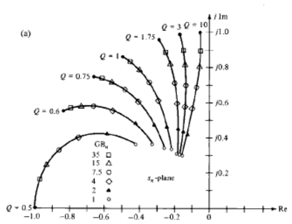




$$T(s)=-K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]}$$

How can the performance of different structures be compared in general?

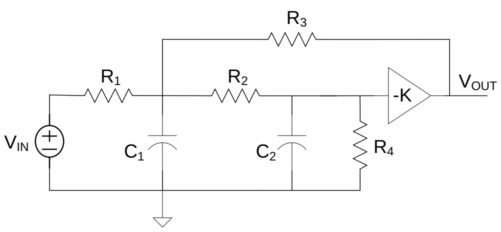


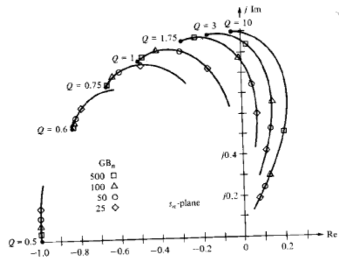


$$T(s)=K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$

$$Q=\frac{1}{\sqrt{\frac{R_2C_2}{R_1C_1}}+\sqrt{\frac{R_1C_2}{R_2C_1}}+(1-K)\sqrt{\frac{R_1C_1}{R_2C_2}}}$$

$$\omega_0=\frac{1}{\sqrt{R_1R_2C_1C_2}}$$





$$T(s)=-K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]}$$

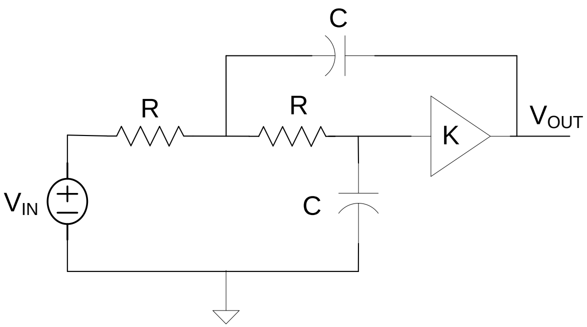
$$\omega_0=\sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}}$$

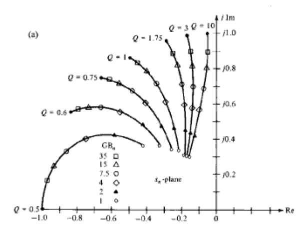
$$Q=\frac{\sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}}}{\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)}$$

- **Equations for key performance parameters give little insight into the differences**
- **Expressions for key performance parameters quite complicated**

How can the performance of different structures be compared in general?

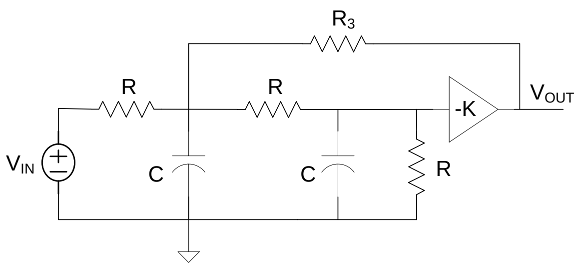
Equal R, Equal C implementations

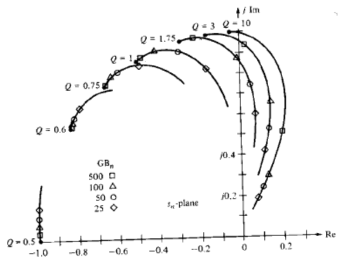




$$T(s)=K\frac{\frac{1}{R^2C^2}}{s^2+s\left[\frac{(3-K)}{RC}\right]+\frac{1}{R^2C^2}}$$

$$Q=\frac{1}{3-K} \qquad \omega_0=\frac{1}{RC}$$



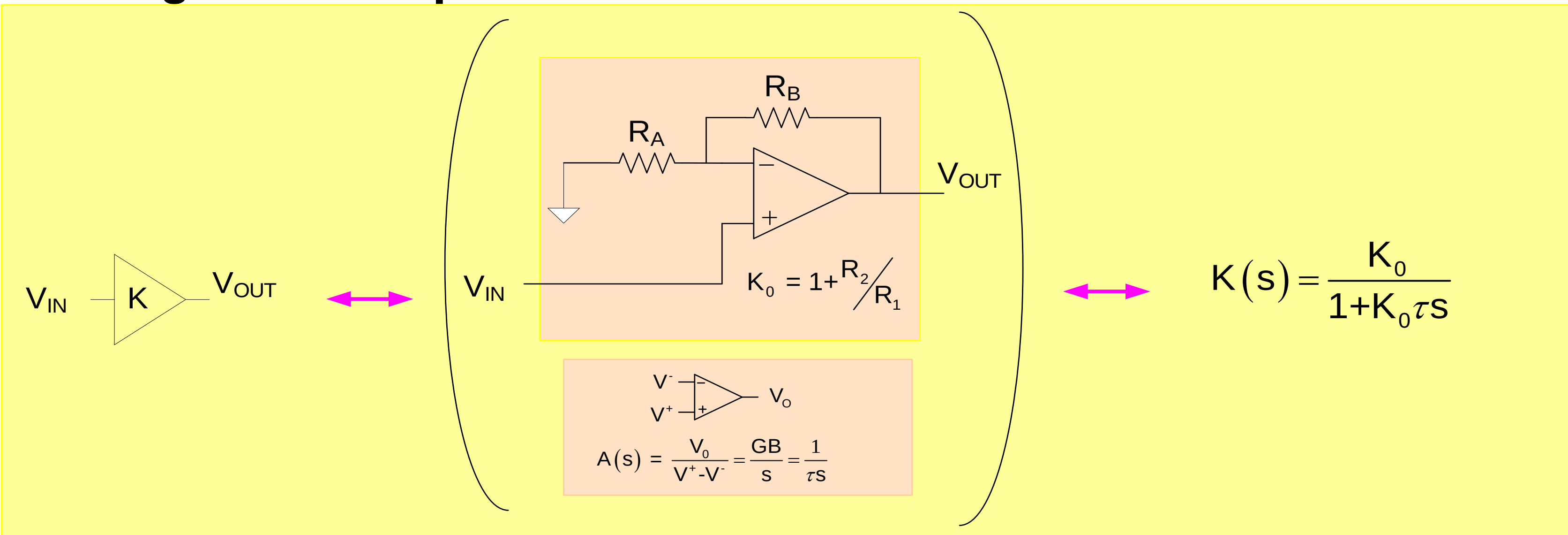


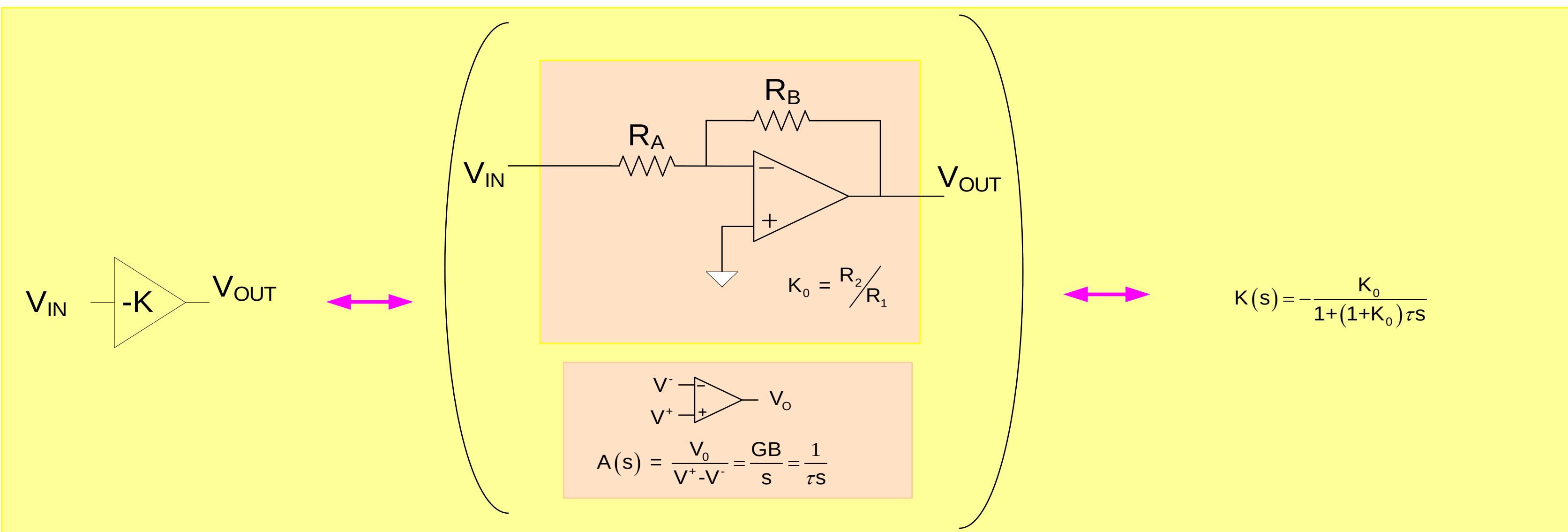
$$T(s)=-K\frac{\frac{1}{R^2C^2}}{s^2+s\left[\frac{5}{RC}\right]+\left[\frac{5+K}{R^2C^2}\right]}$$

$$Q=\frac{\sqrt{5+K}}{5} \qquad \omega_0=\frac{\sqrt{5+K}}{RC}$$

- **Analytical expressions for ω_0 and Q much simpler**
- **Equations for key performance parameters give little insight into the differences**
- **Effects of individual components is obscured in these expressions**
- **GB effects absent in this analytical formulation !!!!**

Modeling of the Amplifiers

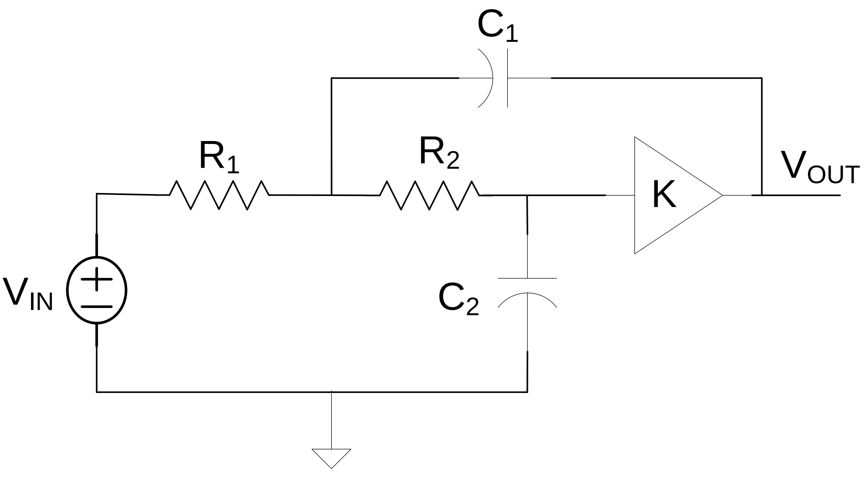




Different implementations of the amplifiers are possible

Have used the op amp time constant in these models $\tau = GB^{-1}$

GB effects in +KRC Lowpass Filter



$$T(s)=K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$

$$K(s)=\frac{K_0}{1+K_0\tau s}$$

$$T(s)=\frac{\frac{K_0}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}+K_0\tau s\left(s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}\right)}$$

$$\omega_0=\frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$Q=\frac{1}{\sqrt{\frac{R_2C_2}{R_1C_1}}+\sqrt{\frac{R_1C_2}{R_2C_1}}+(1-K)\sqrt{\frac{R_1C_1}{R_2C_2}}}$$

ω_0 and Q in these expressions are for ideal op amp

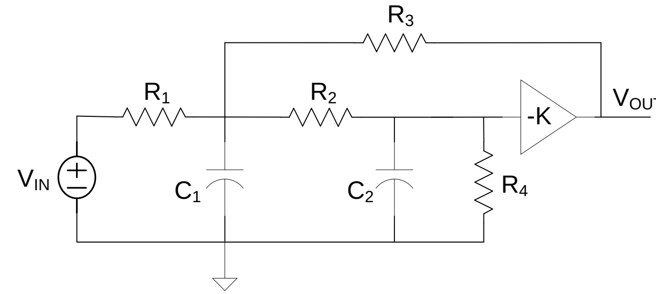
$$T(s)=\frac{K_0\omega_0^2}{s^2+s\left[\frac{\omega_0}{Q}\right]+\omega_0^2+K_0\tau s\left(s^2+s\left[\frac{\omega_0}{Q}\left(1+K_0Q\sqrt{\frac{R_1C_1}{R_2C_2}}\right)\right]+\omega_0^2\right)}$$

$$T(s)=\frac{\frac{K_0}{R_1R_2C_1C_2}}{D_I(s)+K_0\tau s\left(D_{RC0}(s)\right)}$$

$D_I(s)$ is the D(s) if the OA is ideal

$D_{RC0}(s)$ is the D(s) of RC circuit with K=0

GB effects in -KRC Lowpass Filter



$$T(s) = -K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]}$$

$$K(s)=\frac{-K_0}{1+(1+K_0)\tau s}$$

$$Q=\frac{\sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}}}{\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)}$$

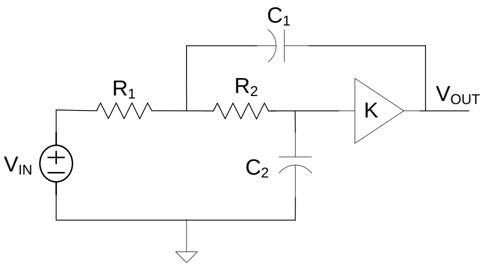
$$\omega_0=\sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}}$$

ω_0 and Q in these expressions are for ideal op amp

$$T(s) = -K_0\frac{\frac{1}{R_1R_2C_1C_2}}{\left(s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)(1+K_0)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]\right) +\tau s(1+K_0)\left(s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]\right)}$$

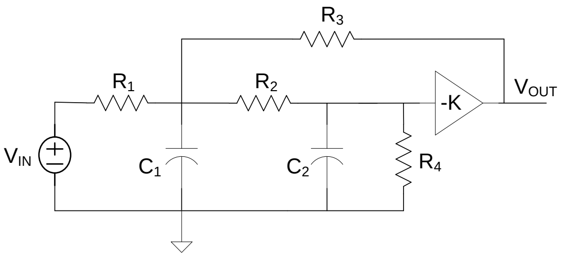
$$T(s)=\frac{\frac{-K_0}{R_1R_2C_1C_2}}{D_I(s)+(1+K_0)\tau s\left(D_{RC0}(s)\right)}$$

GB effects in KRC and -KRC Lowpass Filter



$$T(s)=\frac{K_0\omega_0^2}{s^2+s\left[\frac{\omega_0}{Q}\right]+\omega_0^2+K_0\tau s\left(s^2+s\left[\frac{\omega_0}{Q}\left(1+K_0Q\sqrt{\frac{R_1C_1}{R_2C_2}}\right)\right]+\omega_0^2\right)}$$

$$T(s)=\frac{\frac{K_0}{R_1R_2C_1C_2}}{D_I(s)+K_0\tau s\left(D_{RC0}(s)\right)}$$



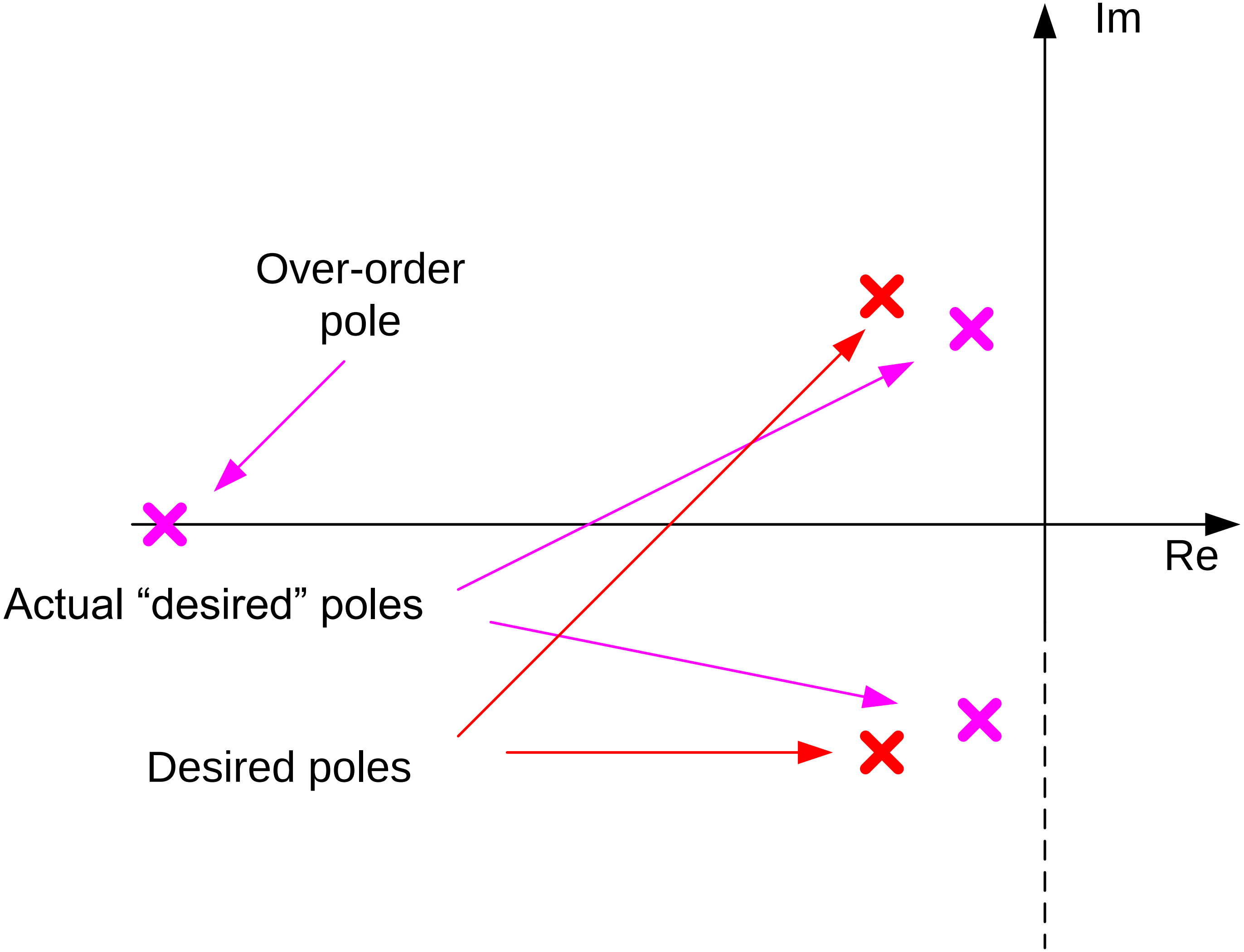
$$T(s)=-K_0\frac{\frac{1}{R_1R_2C_1C_2}}{\left(s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)(1+K_0)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]\right)+\tau s(1+K_0)\left(s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]\right)}$$

$$T(s)=\frac{\frac{-K_0}{R_1R_2C_1C_2}}{D_I(s)+(1+K_0)\tau s\left(D_{RC0}(s)\right)}$$

All linear performance effects can be obtained from this formulation

Op amp introduced an additional pole and moves the desired poles

Effects of GB on poles of KRC and -KRC Lowpass Filters



GB effects in KRC and -KRC Lowpass Filter

$$T(s)=\frac{K_0\omega_0^2}{s^2+s\left[\frac{\omega_0}{Q}\right]+\omega_0^2+K_0\tau s\left(s^2+s\left[\frac{\omega_0}{Q}\left(1+K_0Q\sqrt{\frac{R_1C_1}{R_2C_2}}\right)\right]+\omega_0^2\right)}$$

$$T(s)=-K_0\frac{\frac{1}{R_1R_2C_1C_2}}{\left(s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)(1+K_0)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]\right)+\tau s(1+K_0)\left(s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]\right)}$$

- Analytical expressions for ω_0, Q, poles, zeros, and other key parameters are unwieldly in these circuits and as bad or worse in many other circuits (require solution of 3rd order polynomial!!)
- Sensitivity metrics give considerable insight into how filters perform and are widely used to assess relative performance
- Need sensitivity characterization of real numbers as well as complex quantities such as poles and zeros
- If sensitivity expressions are obtained for a given structure, it can be catalogued rather than recalculated
- **Since analytical expressions for key parameters are unwieldly in even simple circuits, obtaining expressions for the purpose of calculating sensitivity appears to be a formidable task !**

Sensitivity Characterization of Filter Structures

Let F be a filter characteristic of interest

F might be ω_0 or Q of a pole or zero, a band edge, a peak frequency, a BW, T(s), |T(jω)|, a coefficient in T(s), etc

Can express F in terms of all components and model parameters as

$$F=f(R_1, \ldots R_{k1}, C_1, \ldots C_{k2}, L_{l1},\ldots L_{lk3}, \tau_1, \ldots \tau_{k4}, W_1,\ldots W_{k5}, L_1, \ldots L_{k5},\ldots.)$$

$$F=f(x_1,x_2, \ldots x_k)$$

The differential dF of the multivariate function F can be expressed as

$$\begin{aligned} dF &= \frac{\partial F}{\partial R_1}dR_1 + \frac{\partial F}{\partial R_2}dR_2 + \ldots. + \frac{\partial F}{\partial R_{k1}}dR_{k1} \\ &+ \frac{\partial F}{\partial C_1}dC_1 + \frac{\partial F}{\partial C_2}dC_2 + \ldots. + \frac{\partial F}{\partial C_{k2}}dRC_{k2} \\ &+ \ldots\ldots.. \end{aligned}$$

$$dF = \sum_{i=1}^{k} \frac{\partial F}{\partial x_i} dx_i$$

Define the standard sensitivity function as

$$S_x^f = \frac{\partial f}{\partial x} \bullet \frac{x}{f}$$

S_x^f Is widely used except when x or f assume extreme values of 0 or ∞

Define the derivative sensitivity function as

$$\mathcal{S}_x^f = \frac{\partial f}{\partial x}$$

$\mathcal{S}_x^f$ Is more useful when x or f ideally assume extreme values of 0 or ∞

Consider the normalized differential $\dfrac{dF}{F}$

$$\frac{dF}{F} \cong \frac{\Delta F}{F}$$

This approximates the relative (percent if multiply by 100) change in F due to changes in ALL components

$$\frac{dF}{F} = \frac{\sum_{i=1}^{k} \frac{\partial F}{\partial x_i} dx_i}{F} = \sum_{i=1}^{k} \frac{\partial F}{\partial x_i} \bullet \frac{dx_i}{F} \overset{\text{All } x_i \neq 0,\infty}{=} \sum_{i=1}^{k} \left(\frac{\partial F}{\partial x_i} \bullet \frac{x_i}{F} \right) \bullet \frac{dx_i}{x_i}$$

This can be expressed in terms of the standard sensitivity function as

$$\frac{dF}{F} \overset{\text{All } x_i \neq 0,\infty}{=} \sum_{i=1}^{k} \left(S_{x_i}^{f} \bullet \frac{dx_i}{x_i} \right)$$

This relates the relative (percent if multiply by 100) change in F to the sensitivity function and the relative (percent if multiply by 100) change in each component

Consider the normalized differential

$$\frac{dF}{F} = \sum_{i=1}^{k} \left(S_{x_i}^{f} \bullet \frac{dx_i}{x_i} \right)$$

This can be expressed as

$$\frac{dF}{F} = \left(\sum_{all\ resistors} S_{R_i}^{f} \bullet \frac{dR_i}{R_i} \right) + \left(\sum_{all\ capacitors} S_{C_i}^{f} \bullet \frac{dC_i}{C_i} \right) + \left(\sum_{all\ opamps} S_{\tau_i}^{f} \bullet \frac{d\tau_i}{\tau_i} \right) + \ldots$$

Often interested in $\frac{dF}{F}$ evaluated at the ideal (or nominal value)

If the nominal values are all not extreme (0 or ∞), then

$$\frac{dF}{F} = \sum_{i=1}^{k} \left(S_{x_i}^{f} \Big|_{\vec{X}_N} \bullet \frac{dx_i}{x_{iN}} \right)$$

The normalized differential – a different perspective

$$\frac{dF}{F} = \sum_{i=1}^{k} \left(S_{x_i}^{f} \Big|_{\vec{X}_N} \bullet \frac{dx_i}{x_{iN}} \right)$$

Consider the multivariate Taylors series expansion of F

$$F(\vec{X}) = F(\vec{X}_N) + \sum_{i=1}^{k} \frac{\partial F}{\partial x_i}\Big|_{\vec{X}_N} (x_i - x_{iN}) + \left[\frac{1}{2!} \sum_{i=1}^{k} \frac{\partial^2 F}{\partial x_i^2}\Big|_{\vec{X}_N} (x_i - x_{iN})^2 + \sum_{\substack{i=1\\ j=1\\ i \neq j}}^{k,k} \frac{\partial^2 F}{\partial x_i \partial x_j}\Big|_{\vec{X}_N} (x_i - x_{iN})(x_j - x_{jN}) \right] + \ldots\ldots$$

$$F(\vec{X}) \cong F(\vec{X}_N) + \sum_{i=1}^{k} \frac{\partial F}{\partial x_i}\Big|_{\vec{X}_N} (x_i - x_{iN})$$

$$F(\vec{X}) - F(\vec{X}_N) \cong \sum_{i=1}^{k} \frac{\partial F}{\partial x_i}\Big|_{\vec{X}_N} (x_i - x_{iN})$$

$$\Delta F(\vec{X}) \cong \sum_{i=1}^{k} \frac{\partial F}{\partial x_i}\Big|_{\vec{X}_N} \Delta x_i$$

The normalized differential – a different perspective

$$\frac{dF}{F} = \sum_{i=1}^{k} \left(S^{f}_{x_i} \Big|_{\vec{X}_N} \bullet \frac{dx_i}{x_{iN}} \right)$$

Consider the multivariate Taylors series expansion of F

$$\Delta F\left(\vec{X}\right) \cong \sum_{i=1}^{k} \frac{\partial F}{\partial x_i}\bigg|_{\vec{X}_N} \Delta x_i$$

$$\frac{\Delta F\left(\vec{X}\right)}{F} \cong \sum_{i=1}^{k} \frac{\partial F}{\partial x_i}\bigg|_{\vec{X}_N} \frac{\Delta x_i}{F} = \sum_{i=1}^{k} \frac{\partial F}{\partial x_i}\bigg|_{\vec{X}_N} \frac{x_i}{x_i}\frac{\Delta x_i}{F} = \sum_{i=1}^{k} \left(\frac{\partial F}{\partial x_i}\bigg|_{\vec{X}_N} \frac{x_i}{F} \right) \frac{\Delta x_i}{x_i}$$

$$\frac{\Delta F}{F} \cong \sum_{i=1}^{k} \left(S^{f}_{x_i} \Big|_{\vec{X}_N} \right) \frac{\Delta x_i}{x_i}$$

Note this is essentially the same expression that was arrived at from the sensitivity analysis approach

$$\frac{dF}{F} = \sum_{i=1}^{k} \left(S_{x_i}^{f} \Big|_{\vec{X}_N} \bullet \frac{dx_i}{x_{iN}} \right)$$

Dependent only on components (not circuit structure)

Dependent on circuit structure **(for some circuits, also not dependent on components)**

The sensitivity functions are thus useful for comparing different circuit structures

The variability which is the product of the sensitivity function and the normalized component differential is more important for predicting circuit performance

Variability Formulation

$$V_{x_i}^{f} = S_{x_i}^{f}\Big|_{\vec{X}_N} \bullet \frac{dx_i}{x_{iN}}$$

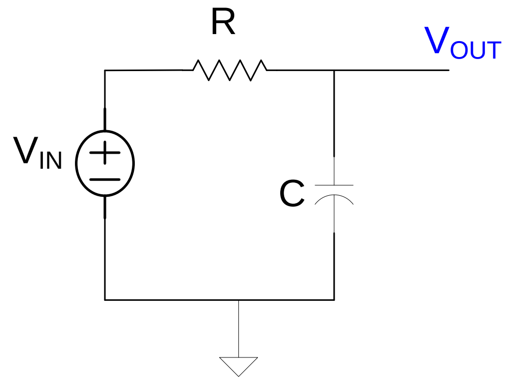
$$\frac{dF}{F} = \sum_{i=1}^{k} V_{x_i}^{f}\Big|_{\vec{X}_N}$$

Variability includes effects of both circuit structure and components on performance

If component variations are small, high sensitivities are acceptable

If component variations are large, low sensitivities are usually critical

Example



$$T(s) = \frac{1}{1+RCs} = \frac{\omega_0}{s+\omega_0}$$

If $\omega_0 = {}^1\!/\!_{RC}$, determine $S_R^{\omega_0}$ and $S_C^{\omega_0}$

$$S_R^{\omega_0} = \frac{\partial \omega_0}{\partial R} \bullet \frac{R}{\omega_0}$$

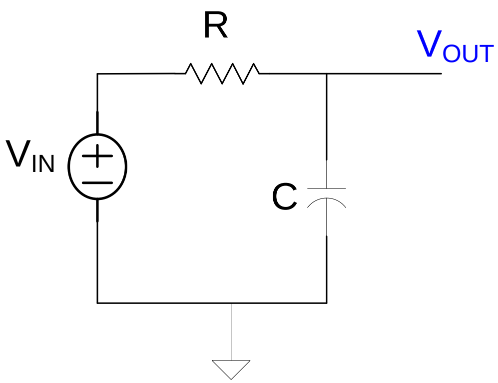
$$S_R^{\omega_0} = \left(\frac{-1}{R^2C}\right) \bullet \frac{R}{\omega_0}$$

$$S_R^{\omega_0} = -\frac{1}{R}\left(\frac{1}{RC}\right) \bullet \frac{R}{\omega_0} = -\frac{1}{R}(\omega_0) \bullet \frac{R}{\omega_0} = -1$$

Likewise

$$S_C^{\omega_0} = -1$$

Example



$$T(s) = \frac{1}{1+RCs} = \frac{\omega_0}{s+\omega_0}$$

$$\omega_0 = 1/RC$$

$$S_R^{\omega_0} = -1 \qquad S_C^{\omega_0} = -1$$

$$\frac{d\omega_0}{\omega_0} = \sum_{i=1}^{k} V_{x_i}^{\omega_0}\Big|_{\vec{X}_N} \qquad V_{x_i}^{f} = S_{x_i}^{f}\Big|_{\vec{X}_N} \bullet \frac{dx_i}{x_{iN}}$$

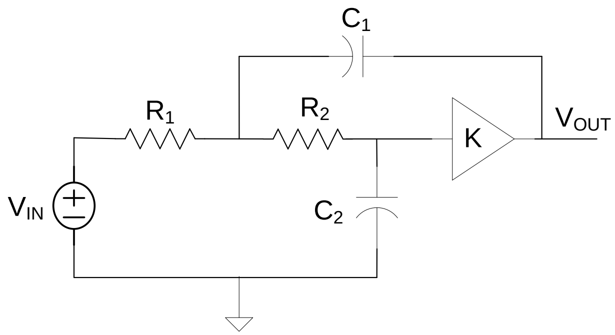
Thus a 1% increase in R will cause approximately a 1% decrease in ω_0

a 1% increase in C will cause approximately a 1% decrease in ω_0

a 1% increase in both C and R will cause approximately a 2% decrease in ω_0

Example

$$T(s)=K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$



$$Q=\frac{1}{\sqrt{\frac{R_2C_2}{R_1C_1}}+\sqrt{\frac{R_1C_2}{R_2C_1}}+(1-K)\sqrt{\frac{R_1C_1}{R_2C_2}}}$$

$$\omega_0=\frac{1}{\sqrt{R_1R_2C_1C_2}}$$

Determine $S_{C_1}^{\omega_0}\quad S_{C_2}^{\omega_0}\quad S_{R_1}^{\omega_0}\quad S_{R_2}^{\omega_0}$

$$S_{C_1}^{\omega_0}=\frac{\partial\left[\frac{1}{\sqrt{R_1R_2C_1C_2}}\right]}{\partial C_1}\;\frac{C_1}{\omega_0}$$

$$S_{C_1}^{\omega_0}=\frac{1}{\sqrt{R_1R_2C_2}}\;\frac{\partial\left[\frac{1}{\sqrt{C_1}}\right]}{\partial C_1}\;\frac{C_1}{\omega_0}$$

$$S_{C_1}^{\omega_0}=\frac{1}{\sqrt{R_1R_2C_2}}\left(-{}^{1}\!/_{2}\,C_1^{-3/2}\right)\;\frac{C_1}{\omega_0}$$

$$S_{C_1}^{\omega_0}=-{}^{1}\!/_{2}\,\frac{1}{\sqrt{R_1R_2C_2}}\left(\frac{1}{\sqrt{C_1}C_1}\right)\;\frac{C_1}{\omega_0}$$

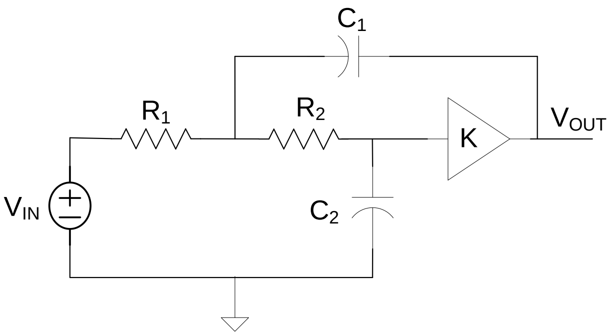
$$S_{C_1}^{\omega_0}=-{}^{1}\!/_{2}\,\frac{1}{\sqrt{R_1R_2C_2C_1}}\left(\frac{1}{C_1}\right)\;\frac{C_1}{\omega_0}$$

$$S_{C_1}^{\omega_0}=-{}^{1}\!/_{2}\,\omega_0\left(\frac{1}{C_1}\right)\;\frac{C_1}{\omega_0}$$

$$S_{C_1}^{\omega_0}=-{}^{1}\!/_{2}$$

Example

$$T(s)=K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$



$$Q=\frac{1}{\sqrt{\frac{R_2C_2}{R_1C_1}}+\sqrt{\frac{R_1C_2}{R_2C_1}}+(1-K)\sqrt{\frac{R_1C_1}{R_2C_2}}}$$

$$\omega_0=\frac{1}{\sqrt{R_1R_2C_1C_2}}$$

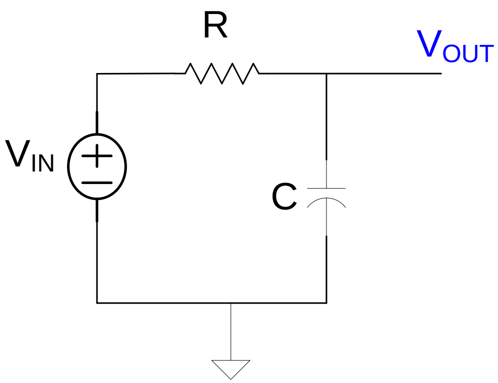
Determine $S_{C_1}^{\omega_0}$ $S_{C_2}^{\omega_0}$ $S_{R_1}^{\omega_0}$ $S_{R_2}^{\omega_0}$

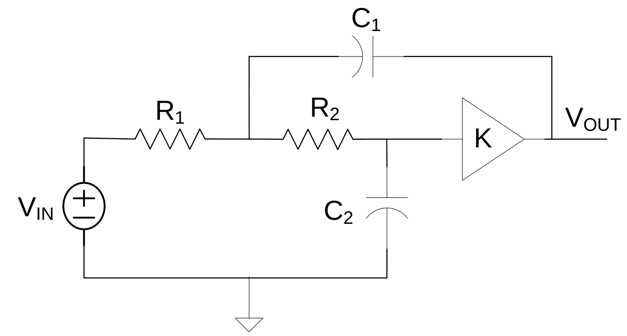
$$S_{C_1}^{\omega_0}=-\tfrac{1}{2}$$

Likewise

$$S_{C_2}^{\omega_0}=S_{R_1}^{\omega_0}=S_{R_2}^{\omega_0}=-\tfrac{1}{2}$$

Observation:





$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$\omega_0 = 1/RC$$

$$S_{R_1}^{\omega_0} = -\tfrac{1}{2} \qquad S_{C_1}^{\omega_0} = -\tfrac{1}{2}$$

$$S_R^{\omega_0} = -1 \qquad S_C^{\omega_0} = -1$$

$$S_{R_2}^{\omega_0} = -\tfrac{1}{2} \qquad S_{C_2}^{\omega_0} = -\tfrac{1}{2}$$

$$\sum_{All\ resistors} S_{R_i}^{\omega_0} = -1 \qquad \sum_{All\ capacitors} S_{C_i}^{\omega_0} = -1$$

$$\sum_{All\ resistors} S_{R_i}^{\omega_0} = -1 \qquad \sum_{All\ capacitors} S_{C_i}^{\omega_0} = -1$$

At this stage, this is just an observation about summed sensitivities but later will establish some fundamental properties of summed sensitivities

Consider

$$\frac{dF}{F}=\left(\sum_{all\ resistors} S^f_{R_i}\bullet\frac{dR_i}{R_i}\right)+\left(\sum_{all\ capacitors} S^f_{C_i}\bullet\frac{dC_i}{C_i}\right)+\left(\sum_{all\ opamps} S^f_{\tau_i}\bullet\frac{d\tau_i}{\tau_i}\right)+\ldots$$

The nominal value of the time constant of the op amps is 0 so this expression can not be evaluated at the ideal (nominal) value of GB=∞

Let {xi} be the components in a circuit whose nominal value is not 0

Let {yi} be the components in a circuit whose nominal value is 0

$$\frac{dF}{F}=\sum_{i=1}^{kx}\frac{\partial F}{\partial x_i}\bullet\frac{dx_i}{F}+\sum_{i=1}^{ky}\frac{\partial F}{\partial y_i}\bullet\frac{dy_i}{F}=\sum_{i=1}^{k}\left(\frac{\partial F}{\partial x_i}\bullet\frac{x_i}{F}\right)\bullet\frac{dx_i}{x_i}+\frac{1}{F}\sum_{i=1}^{ky}\frac{\partial F}{\partial y_i}dy_i$$

$$\frac{dF}{F}=\sum_{i=1}^{k}\left(S^f_{X_i}\Big|_{\vec{X}_N,\vec{Y}_N=0}\bullet\frac{dx_i}{x_i}\right)+\frac{1}{F_N}\sum_{i=1}^{ky}\left(\mathcal{S}^f_{y_i}\Big|_{\vec{X}_N,\vec{Y}_N=0}\bullet y_i\right)$$

This expression can be used for predicting the effects of all components in a circuit

Can set Y_N=0 before calculating $S^f_{x_i}$ **functions**

$$\frac{dF}{F}=\sum_{i=1}^{k}\left(S_{x_i}^{f}\Big|_{\vec{X}_N}\bullet\frac{dx_i}{x_i}\right)+\frac{1}{F_N}\sum_{i=1}^{ky}\left(\mathcal{S}_{y_i}^{f}\Big|_{\vec{Y}_N=0}\bullet y_i\right)$$

Low sensitivities in a circuit are often preferred but in some applications, low sensitivities would be totally unacceptable

Examples where low sensitivities are unacceptable are circuits where a charactristics F must be tunable or adjustable!

Some useful sensitivity theorems

$$S_x^{kf} = S_x^f$$

where k is a constant

$$S_x^{f^n} = n \bullet S_x^f$$

$$S_x^{1/f} = -S_x^f$$

$$S_x^{\sqrt{f}} = \frac{1}{2} S_x^f$$

$$S_x^{\prod_{i=1}^{k} f_i} = \sum_{i=1}^{k} S_x^{f_i}$$

Some useful sensitivity theorems (cont)

$$S_x^{f/g} = S_x^f - S_x^g$$

$$S_x^{\sum_{i=1}^{k} f_i} = \frac{\sum_{i=1}^{k} f_i S_x^{f_i}}{\sum_{i=1}^{k} f_i}$$

$$S_{1/x}^{f} = -S_x^f$$

Stay Safe and Stay Healthy !

End of Lecture 18